ME5331 Homework #4
Due 11/5/03

1. If $\phi = x^2 yz^3$ and $\vec{A} = xz\vec{i} - y^2\vec{j} + 2x^2 y\vec{k}$, find (a) $\nabla\phi$, (b) $\nabla\cdot\vec{A}$, (c) $\nabla\times\vec{A}$, (d) $\nabla\cdot(\phi\vec{A})$, (e) $\nabla\times(\phi\vec{A})$.

2. Find a unit normal to the surface $2x^2 + 4yz - 5z^2 = -10$ at the point P(3,-1,2).

3. (a) Find the directional derivative of $U = 2xy - z^2$ at (2,-1,1) in a direction toward (3,1,-1).
   (b) In what direction is the directional derivative a maximum?
   (c) What is the value of this maximum?

4. Evaluate $\int_{(0,1)}^{(1,2)}[(x^2 - y)dx + (y^2 + x)dy]$ along
   (a) a straight line from(0,1) to (1,2),
   (b) straight lines from(0,1) to (1,1) and then from (1,1) to (1,2),
   (c) the parabola x=t, y=t^2+1.

5. Evaluate $\iint_S [xz^2 dydz + (x^2 y - z^3)dzdx + (2xy + y^2 z)dxdy]$ where S is the entire surface of the hemispherical region bounded by $z = \sqrt{a^2 - x^2 - y^2}$ and z=0.

6. (a) Prove that $\vec{F} = (2xz^3 + 6y)\vec{i} + (6x - 2yz)\vec{j} + (3x^2 z^2 - y^2)\vec{k}$ is a conservative force field.
   (b) Evaluate $\int_C \vec{F}\cdot d\vec{r}$ where C is any path from (1,-1,1) to (2,1,-1).