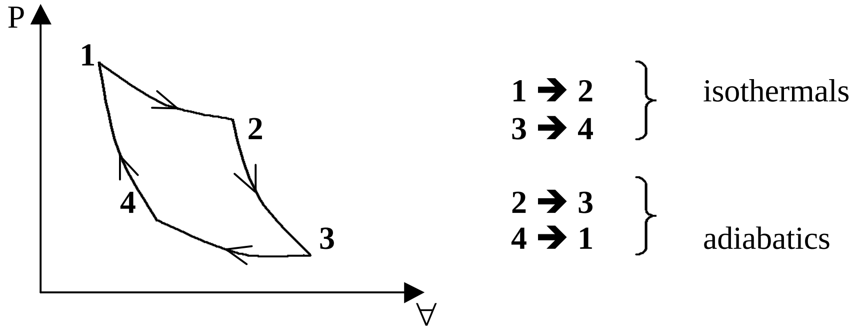
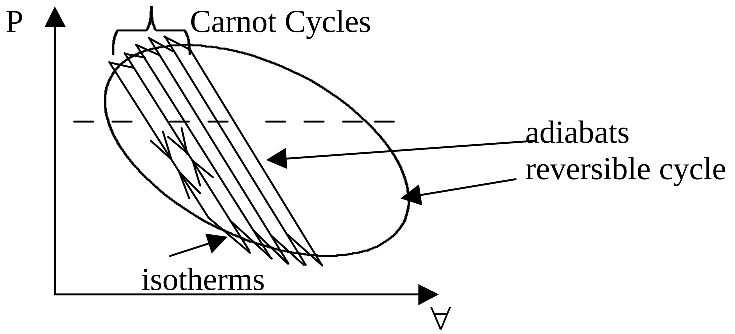
CHAPTER 7: ENTROPY

7.1: Entropy a Thermodynamic Property

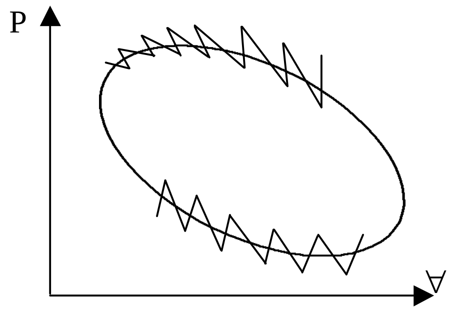
In general, a P-∀ plot for a Carnot Cycle is given by:



It is possible to represent a reversible cycle by a series of Carnot Cycles. Consider the following reversible cycle with a series of Carnot Cycles added:



The isotherms and part of the adiabats of the Carnot Cycles form a zigzag curved (the portion of the adiabats which is shared by adjacent cycles will cancel each other because each section is traversed once in a forward direction and in in a revered direction) which follows closely the path of the original cycle



As the number of Carnot Cycles is increased the zigzag curve can be made to approach the reversible cycle to any desires degree of accuracy

For each small individual Carnot Cycle, we have:

$$\frac{dQ_H^i}{T_H^i} + \frac{dQ_L^i}{T_L^i} = 0$$

where i denotes the ith Carnot Cycle. Therefore,

$$\sum_{i=1}^{N} \left(\frac{dQ_H^i}{T_H^i} + \frac{dQ_L^i}{T_L^i} \right) = 0$$

where N is the total number of Carnot Cycle

$$\oint \frac{dQ}{T} \le 0 \qquad (1) \qquad\qquad \oint \frac{dQ}{T} = 0 \qquad (2)$$

valid for all cycles; valid for reversible cycle

“T”represents the temperature of the source which considers the quantity of heat δQ and is not necessarily equal to the temperature T’ of the system which receives the heat δQ. Indeed, if the cycle is irreversible, T’s ≤ T when δQ is positive because heat cannot flow from a colder body to a hotter body; and when δQ is negative T’ ≥ T. If the cycle is reversible, however, we must always have T’ = T, because an exchange of heat between two bodies at different temperatures is not reversible. In Equation (2) we may therefore take T to be the temperature of the source and also the temperature of the part of the system that receives the heat δQ.

In the limit, as N goes to infinity, we have

$$\oint_{cycle} \frac{dQ}{T} = 0$$

But this is for reversible cycle only, hence we denote that by

$$\boxed{\oint \left(\frac{dQ}{T}\right)_{rev} = 0} \qquad (1)$$

It follows that (see the derivation of the first law for a closed system undergo a cycle the differential $(\frac{dQ}{T})_{rev}$ is an exact different and $\int (\frac{dQ}{T})_{rev}$ is a property of the system (i.e. path independent). This property is called entropy and is denoted by the symbol S. Thus

$$\boxed{dS \equiv \left(\frac{dQ}{T}\right)_{rev}} \qquad (2)$$

Integrating along a reversible path between two equilibrium state (1 and 2) give:

$$\boxed{\Delta S = S_2 - S_1 = \int_1^2 \left(\frac{dQ}{T}\right)_{rev}} \qquad (3)$$

Note: Since entropy is a point function, it does not matter which particular reversible path is followed in the integration as long as it reversible.

According to (2)

$$dQ = Tds \qquad \text{for a reversible process}$$

➔ $${}_1Q_2 = \int_1^2 TdS$$

Therefore the amount of heat transfer in a reversible process is given by the area under the curve on a T vs. S diagram. This is similar to the calculation of expansion work in a quasi-equilibrium process. In that case, the amount of work done is given by the area under the curve on the P vs. ∀ plot.

For a reversible adiabatic process, we have

$$dS = \left(\frac{dQ}{T}\right)_{rev} = 0 \qquad \text{since } \delta Q=0$$

A reversible adiabatic process during which there is no change in entropy is called isentropic process

7.2: Inequality of Clausius:

For any irreversible cycle, we have

$$\oint \left(\frac{dQ}{T}\right)_{irr} < 0$$

This is the *Inequality of Clausius*

Proof:

From Carnot principles, the efficiency of an irreversible engine is smaller than that of a reversible engine operating between the same temperature limits, i.e.

$$h_{irr} < h_{rev}$$

In terms of heat quantities

$$1 - \frac{|dQ_L|_{irr}}{|dQ_H|_{irr}} < 1 - \frac{|dQ_L|_{rev}}{|dQ_H|_{rev}}$$

➔ $$1 - \frac{|dQ_L|_{irr}}{|dQ_H|_{irr}} < 1 - \frac{T_L}{T_H}$$

➔ $$-\frac{|dQ_L|_{irr}}{T_L} + \frac{|dQ_H|_{irr}}{T_H} < 0$$

➔ $$\frac{dQ_{H\,irr}}{T_H} + \frac{dQ_{L_{irr}}}{T_L} < 0$$

Note: $|\delta Q_L|$ is the absolute value of heat rejection, it's algebraic value, therefore has a negative sign according our sign convention. Summing up all cycles (similar to last section) gives:

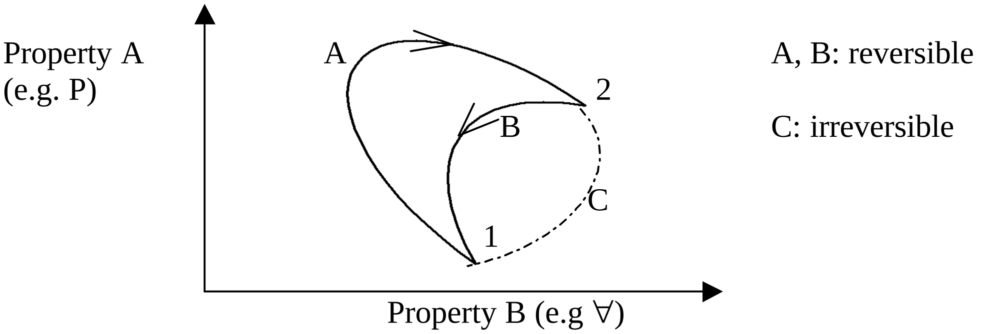
$$\sum_{i=1}^{N} \left(\frac{dQ_H}{T_H}\right)_{irr}^{i} + \left(\frac{dQ_L}{T_L}\right)_{irr}^{i} < 0$$

where N is the total number of individual cycles. Taking the limit N→∞, we have

$$\boxed{\oint \left(\frac{dQ}{T}\right)_{irr} < 0} \qquad \text{Q.E.D}$$

Note: The entropy of a system is a point function, it depends only on the state that the system is in, and not on how that state is reached. For example, if a system goes from state 1 to state 2 its entropy changes from S1 to S2. It is only when the system travels along a reversible path between the two end states that the entropy change S2 – S1, equal (equation). If the path is irreversible, (equation) has a different value although the change in entropy is the same as before (since entropy is a point function)

Consider the following:



(i) 1A2B1: reversible cycle

$$\int_{1,A}^{2} \frac{dQ}{T} + \int_{2,B}^{1} \frac{dQ}{T} = 0$$

(ii) 1A2C1: irreversible cycle

$$\int_{1,A}^{2} \frac{dQ}{T} + \int_{2,C}^{1} \frac{dQ}{T} < 0$$

Eq (2) – Eq(1) gives:

$$\int_{2,C}^{1} \frac{dQ}{T} - \int_{2,B}^{1} \frac{dQ}{T} < 0$$

$$\Rightarrow \qquad \int_{2,B}^{1} \frac{dQ}{T} > \int_{2,C}^{1} \frac{dQ}{T}$$

But $$\int_{2,B}^{1} \frac{dQ}{T} = \int_{2}^{1} dS$$; since $dS = (\frac{dQ}{T})_{rev}$

and $$\int_{2,B}^{1} dS = \int_{2,C}^{1} dS = \int_{2}^{1} dS$$

(since S is a property)

$$\therefore \qquad \int_{2}^{1} dS > \int_{2,C}^{1} \frac{dQ}{T}$$

or $$\boxed{dS > (\frac{dQ}{T})_{irr}}$$

or $$\boxed{\Delta S = S_2 - S_1 > \int_{1}^{2} (\frac{dQ}{T})_{irr}}$$

Summary:

(i) $$\oint (\frac{dQ}{T}) \leq 0$$

(ii) $$dS \geq \frac{dQ}{T}$$

where equality hods for reversible process cycle and inequality holds for irreversible process cycle

7.3: The Priciple of Increase of Entropy for an Isolated system

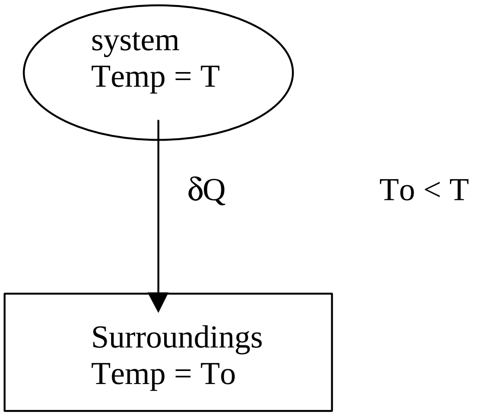
For an isolated system, δQ = 0. From previous results,

$$dS \geq \frac{dQ}{T} = 0$$ for isolated system

or $$\boxed{dS_{isolate} \geq 0}$$

This is the Principle of Increase of Entropy for an isolated system. Those are the only process that can take place are those in which net change in entropy of the system plus its surroundings increases (or in the limit remains constant)

Example: cooling of a higher temperature system by a lower temperature surroundings



$$dS_{system} \geq -\frac{dQ}{T}$$ (since heat is transferred out of the system)

$$dS_{surrounding} \geq \frac{dQ}{T_0}$$

❖ $$dS_{isolatdsystem} = dS_{system} + dS_{surrounding} \geq dQ\left(\frac{1}{T_0} - \frac{1}{T}\right)$$

➔ $$dS_{system} + dS_{surrounding} \geq \frac{-dQ}{T} + \frac{dQ}{T_0}$$

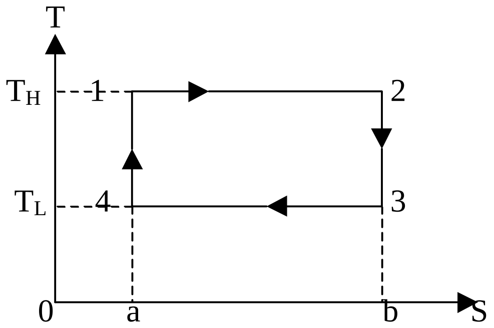
since $T > T_0 \Rightarrow \frac{1}{T_0} - \frac{1}{T} > 0$

$$\boxed{d.S_{isolatedsystem} = dS_{system} + dS_{surrounding} > 0}$$

When an iso lated system is in the state of maximum entropy consistent with its energy it cannot undergo any function transformation because any transformation could result in a decreased of entropy. Thus, the state of maximum entropy is the most static state for an isolated system.

7.4 Entropy Change for a Carnot Cycle

It can be shown that a Carnot Cycle can be represented as a rectangular cycle on the T-S diagram.



1 ➔ 2: reversible isothermal process

$$S_1 - S_2 = \frac{1}{T_H}\int_1^2 dQ = \frac{{}_1Q_2}{H}$$

➔ $${}_1Q_2 = T_H(S_2 - S_1) = Area12ba1$$

2 ➔ 3: reversible adiabatic process

$$S_3 - S_2 = \int_2^3 \frac{dQ}{T} = 0$$ since $\delta Q = 0$

➔ $S_3 = S_2$

3 ➔ 4: reversible isothermal process

$$S_4 - S_3 = \frac{1}{T_L}\int_3^4 dQ = -\frac{{}_3Q_4}{T_L}$$

($-{}_3Q_4$ because heat is transferred from the system to the surrounding)

➔ $${}_3Q_4 = -T_L(S_4 - S_3) = -Area43ba4$$

4 ➔ 1: reversible adiabatic process

$S_4 = S_1$

Recall $$h \equiv \frac{Work-done}{Energy-Input} = \frac{Area12341}{Area12ba1}$$

7.5: Two Important Thermodynamic Relations for a Simple Compressible Substance

1. $Tds = du + Pdv$
2. $Tds = dh - vdP$

Derivation:

1. 1From 1st law: $\delta Q = dU + \delta W$ ….

For reversible process: $\delta Q = TdS$,$\delta W = Pd\forall$

1st law becomes: $TdS = dU + Pd\forall$…… (1)

or $Tds = du + Pdv$ (2)

Note: Even though this relation is derived by assuming that the process is reversible. Since it only involve properties, and properties are point function which are independent of path, the relation is valid for any process. However, it can only be integrated for a reversible process

2. $H = U + P\forall$

➔ $dH = dU + Pd\forall + \forall dP$

sub. into (1) gives: $TdS = dH - \forall dP$

or $Tds = dh - vdP$

7.6: Entropy Change of an Ideal Gas

From the results of section 7.5

$$Tds = du + Pdv \qquad (1)$$

$$Tds = dh - vdP \qquad (2)$$

For ideal gas : $$\left.\begin{array}{l} du = C_{vo}\, dT \\ P = \frac{RT}{v} \end{array}\right\} \qquad (3)$$

and

$$dh = C_{po}\, dT$$

Sub. into (1) gives:

$$TdS = C_{vo}\, dT + \frac{RT}{v}\, dv$$

➔ $$dS = C_{vo}\frac{dT}{T} + R\frac{dv}{v}$$

➔ $$\boxed{s_1 - s_2 = \int_1^2 C_{vo}\frac{dT}{T} + R\ln\frac{v_2}{v_1}} \qquad (4)$$

sub. (3) into (2) gives:

$$Tds = C_{po}\, dT - RT\frac{dP}{P}$$

➔ $$ds = C_{po}\frac{dT}{T} - R\frac{dP}{P}$$

➔ $$\boxed{s_2 - s_1 = \int_1^2 C_{po}\frac{dT}{T} - R\ln\frac{P_2}{P_1}} \qquad (5)$$

Special Case: C_{po} = constant; C_{vo} = constant

Equ. (4) becomes: $$s_2 - s_1 = C_{vo}\ln\frac{T_2}{T_1} + R\ln\frac{v_2}{v_1}$$

Equ (5) becomes: $$s_2 - s_1 = C_{po}\ln\frac{T_2}{T_1} + R\ln\frac{P_2}{P_1}$$

<u>Example:</u>

Air is heated form 540°R to 1200°R while pressure drops form 50 psia to 40 psia. Assume: C_{po} = 0.241 Btu / lbm-°R; R = 53.3 ft-lbf / lbm-°R. Calculate the change in entropy

T1 = 540°R P1 = 50 psia
T2 = 1200°R P2 = 40 psia

$$\Delta S = S_2 - S_1 = C_{po}\ln\frac{T_2}{T_1} - R\ln\frac{P_2}{P_1}$$

$$= 0.241\frac{Btu}{lbm^\circ R}\ln\frac{1200}{540} - \frac{53.3\frac{ft-lbf}{lbm^\circ R}}{778\frac{ft-lbf}{Btu}}\ln\frac{40}{50}$$

$$= 0.207\frac{Btu}{lbm^\circ R}$$

__Note__: the equation used to calculate ΔS consists of properties only and hence path independent. Even though the process is not reversible, thechange in entropy can

7.7: Insetropic Process for an Ideal Gas with Constant Specific Heats

Define specific heat ratio, k, by
$$\boxed{k \equiv \frac{C_{po}}{C_{vo}}} \qquad (1)$$

Recall: $C_{po} - C_{vo} = R$

➔ $\frac{C_{po}}{C_{vo}} - 1 = \frac{R}{C_{co}}$

➔ $$\boxed{C_{vo} = \frac{R}{k-1}} \qquad (2)$$

Also: $C_{po} = C_{vo} + R = \frac{R}{k-1} + R\frac{(k-1)}{(k-1)}$

➔ $$\boxed{C_{po} = \frac{Rk}{k-1}} \qquad (3)$$

__Note__: Although $C_{po} - C_{vo} = R$ is a constant for ideal gas, C_{po} and C_{vo} are function of temperature. It follows that k is also a function of temperature. In the following deviation, C_{po} and C_{vo} and hence k are assumed to be constant.

For an isotropic process, dS = 0

∴ Tds = du + Pdv = 0

➔ du + Pdv = 0

➔ C_{vo} dT = -Pdv….. (4)

But Pv = RT

➔ pdv + vdP = RdT

➔ $dT = \frac{Pdv + vdT}{R}$

Sub. into (4) gives:

$$\frac{C_{vo}}{R}(pdv + vdp) = -pdv$$

➔ $\frac{1}{k-1}(pdv + vdp) = -pdv$

➔ pdv + vdp + kpdv – pdv =0

➔ ln(P) + vdp + kln(U) = constant

➔ $$\boxed{pv^k = const} \qquad (5)$$

Note: This is valid for
- (i) isentropic process, with
- (ii) ideal gas
- (iii) constant specific heats

Form (5) $P_1 v_1^k = P_2 v_2^k = \text{constant}$

$$\Rightarrow \boxed{\frac{P_1}{P_2} = \left(\frac{v_2}{v_1}\right)^k} \qquad (6)$$

From Ideal gas law: Pv = Rt

$$\Rightarrow \frac{P_1}{P_2} = \frac{T_1}{T_2}\frac{v_2}{v_1}$$

Sub. into (6) gives:

$$\Rightarrow \frac{T_1}{T_2}\frac{v_2}{v_1} = \left(\frac{v_2}{v_1}\right)^k \qquad (7)$$

$$\Rightarrow \boxed{\frac{T_1}{T_2} = \left(\frac{v_2}{v_1}\right)^{k-1}}$$

Also $$\frac{v_2}{v_1} = \frac{p_1}{p_2}\frac{T_2}{T_1}$$

Sub. into (7) gives

$$\Rightarrow \frac{T_1}{T_2} = \left(\frac{p_1}{p_2}\right)^{k-1}\left(\frac{T_2}{T_1}\right)^{k-1}$$

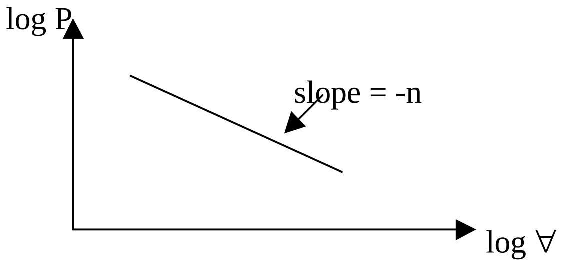
$$\Rightarrow \left(\frac{T_1}{T_2}\right)^k = \left(\frac{p_1}{p_2}\right)^{k-1}$$

$$\Rightarrow \boxed{\frac{T_1}{T_2} = \left(\frac{p_1}{p_2}\right)^{\frac{k-1}{k}}} \qquad (8)$$

7.8 Reversible Polytropic Process for an Ideal Gas

Define: A polytropic process is a process during which $P\forall^n$ = constant (n is called the polytropic constant)

On a P-∀ plot we have



$$P\forall^n = const \Rightarrow P_1\forall_1^{\,n} = P_2\forall_2^{\,n}$$

Similar to the procedure used in the last section, we have (with n replaced by k)

$$\boxed{\left(\frac{P_1}{P_2}\right) = \left(\frac{\forall_2}{\forall_1}\right)^n}$$

$$\boxed{\left(\frac{T_1}{T_2}\right) = \left(\frac{v_2}{v_1}\right)^{n-1}}$$

$$\boxed{\left(\frac{T_1}{T_2}\right) = \left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}}}$$

For a reversible process: $W = \int P d\forall$

For a reversible polytropic process: $P\forall^n = C$

➔ $${}_1W_2 = \int_1^2 \frac{C}{\forall^n} d\forall$$

(i) for $n \neq 1$

$$\begin{aligned} {}_1W_2 &= C \frac{\forall^{-n+1}}{-n+1}\Big|_1^2 \\ &= \frac{C}{1-n}\left[\forall_2^{\,1-n} - \forall_1^{\,1-n}\right] \end{aligned}$$

Now:

$$C = P\forall^n = P_1\forall_1^{\,n} = P_2\forall_2^{\,n}$$

$$\begin{aligned} \therefore \quad {}_1W_2 &= \frac{C\forall_2^{\,1-n} - C\forall_1^{\,1-n}}{1-n} \\ &= \frac{P_2\forall_2^{\,n}\forall_2^{\,1-n} - P_1\forall_1^{\,1-n}\forall_1^{\,1-n}}{1-n} \end{aligned}$$

$$\boxed{{}_1W_2 = \frac{P_2\forall_2 - P_1\forall_1}{1-n}}$$

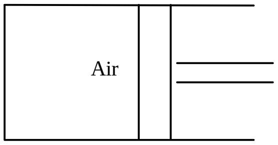
or $$\boxed{{}_1W_2 = -\frac{mR(T_2-T_1)}{1-n}}$$

(ii) for $n = 1$

$${}_1W_2 = \int_1^2 C\frac{d\forall}{\forall} = C\ln\frac{\forall_2}{\forall_1}$$

$$\boxed{{}_1W_2 = P_1\forall_1 \ln\frac{\forall_2}{\forall_1} = P_2\forall_1 \ln\frac{\forall_2}{\forall_1}}$$

7.9: Worked Example



Initially:

$\forall_1 = 10\ ft^3$

$P_1 = 20$ psia ⟹ $P_2 = 80$ psia

$T_1 = 70°F$ (reversible polytropic $P\forall n = C$ $T_2 = 250°R$

Determine: (a) "n" the polytropic constant

(b) $\forall_2$

(c) ${}_1W_2$

(d) ${}_1Q_2$

(e) ΔS_{net}

Assume ideal gas with constant specific heats:

$C_{vo} = 0.171$ Btu / lbm-°R

$C_{vo} = 0.241$ Btu / lbm-°R

R = 53.34 ft-lbf / lbm-°R

(a)

$$m = \frac{P_1\forall_1}{RT_1} = \frac{(20\frac{lbf}{in^2})(144\frac{in^2}{ft^2})(10\ ft^3)}{(53.34\frac{ft-lbf}{lbm-R})(530R)}$$

$$= \boxed{1.02\ \text{lbm}}$$

$$v_1 = \frac{\forall_1}{m} = \frac{10\ ft^3}{1.02 lbm} = 9.8\frac{ft^3}{lbm}$$

$$v_2 = \frac{RT_2}{P_2} = \frac{(53.34)(710)}{(80)(144)} = 3.28\frac{ft^3}{lbm}$$

$$p_1v_1^{\ n} = p_2v_2^{\ n}$$

➔

$$\boxed{n = \frac{\ln(\frac{p2}{p1})}{\ln(\frac{v1}{v2})} = \frac{\ln(80/20)}{\ln(9.8/3.28)} = 1.27}$$

(b) $\forall_2 = mv_2 = (1.02)(3.28) = \boxed{3.35\ ft^3}$

(c)

$${}_1W_2 = \frac{p_2\forall_2 - p_1\forall_1}{1-n} = \frac{mR(T_2 - T_1)}{1-n}$$

$$\boxed{= \frac{(1.02)(53.34)(710-530)}{(1-1.27)(778)} = -46.6 Btu}$$

(d)

$$
\begin{aligned}
{}_1Q_2 &= U_2 - U_1 + {}_1W_2 \\
&= m(u_2 - u_1) + {}_1W_2 \\
&= C_{vo}(T_2 - T_1) + {}_1W_2 \\
&= (1.02)(0.171)(710\text{-}530) + (-46.6) \\
&= \boxed{-15.2 \text{ Btu}}
\end{aligned}
$$

(e)

$$\Delta S_{net} = \Delta S_{system} + \Delta S_{sum}$$

$$\Delta_{system} = m[C_{po} \ln \tfrac{T_2}{T_1} - R \ln \tfrac{P_2}{P_1}]$$

$$= (1.02)[0.24 \ln(\tfrac{710}{530}) - \tfrac{53.34}{778} \ln(\tfrac{80}{20})$$

$$= -0.0251 \tfrac{Btu}{{}^\circ R}$$

$$\Delta S_{surr} = -\frac{{}_1W_2}{T_0} = \frac{15.2}{530} = 0.0297 \tfrac{Btu}{{}^\circ R}$$

∴

$$\boxed{\Delta S_{net} = \Delta S_{system} + \Delta S_{sum} = 0.0046 \tfrac{Btu}{{}^\circ R}}$$

ξ **Entropy Change of a Solid or Liquid**

Recall, for either a solid or a liquid

$$dh \approx du \approx C\, dT \qquad (1)$$

where C is either Cp or Cv, as the two would be nearly the same. Interpolating gives:

$$h2 - h1 \approx u2 - u1 \approx C\,(T2 - T1) \qquad (2)$$

From one of the two thermodynamic relation:

$$Tds = du + Pdv \qquad (3)$$

since dv ≈ 0 for solid and liquid, we have

$$Tds \approx du$$

➔ $$ds \approx \frac{du}{T}$$

sub. (1) for du gives:

$$ds \approx \frac{CdT}{T} \qquad (4)$$

For many process involving liquid and solid. C remains constant, (4) because after interpolation

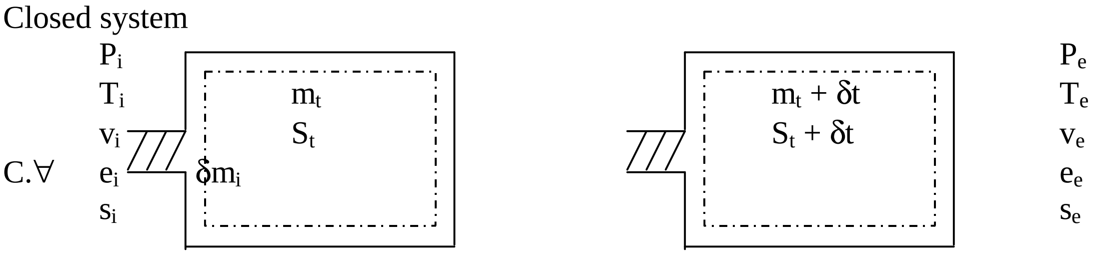
$$\boxed{S_2 - S_1 \approx C \ln \tfrac{T_2}{T_1}} \qquad (5)$$

7.10 The Second Law of Thermodynamics for a Control Volume

For a closed system, 2nd law says:

$$dS \geq \frac{dQ}{T}$$

Consider the closed system along with the C.∀ shown below: (similar to the C.∀ analysis of chapter 5)



at time t at time t +δt

Notations :

m_t	mass in C.∀ at time t
$m_t + \delta t$	mass in C.∀ at time t + δt
S_t	entropy of system at time t
$S_t + \delta t$	entropy of system at time t + δt
m_1	mass of the system at time t
m_2	mass of the system at time t + δt
S_1	entropy of system at time t
S_2	entropy of system at time t + δt
δQ	heat transfer into the C.∀

$$S_2 - S_1 \geq \frac{dQ}{T} \qquad (1)$$

$m_1 = m_t + \delta m_i$
$m_2 = m_{t+\delta t} + \delta m_e$
$S_1 = S_{t\,+\,\delta} + S_i m_i$
$S_2 = S_{t+\delta t} + S_e m_e$

L.H.S of (1):

$$S_1 - S_2 = (S_{t+\delta t} - S_t) + S_e \delta m_e - S_i \delta m_i$$

R.H.S of (1):

$$\frac{dQ}{T} = \Big(\iint_{c.s.} \frac{\dot{q}''}{T} dA\Big)dt$$

Equ. (1) become:

$$S_{t+dt} - S_t + s_e dm_e - s_i dm_i \geq \Big(\iint_{c.s.} \frac{\dot{q}''}{T} dA\Big)dt$$

Dividing by δt gives:

$$\frac{S_{t+dt}-S_t}{dt} + s_e\,\frac{dm_e}{dt} - s_i\,\frac{dm_i}{dt} \geq \iint_{c.s.} \frac{q^{\cdot''}}{T}\,dA$$

taking the limit δt → 0 gives:

$$\boxed{\frac{dS_{c.v}}{dt} + m^{\cdot}{}_{e}\,s_e - m^{\cdot}{}_{i}\,s_i \geq \iint_{c.s.} \frac{q^{\cdot''}}{T}\,dA}$$

This is the second law of thermodynamics for a control volume with single inlet and single exit. For multiple inlets and multiple exits, we have,

$$\boxed{\frac{dS_{c.v}}{dt} + \sum_{exit} m^{\cdot}{}_{e}\,s_e - \sum_{inlet} m^{\cdot}{}_{i}\,s_i \geq \iint_{c.s.} \frac{q^{\cdot''}}{T}\,dA}$$

7.11 SSSF Process Reverted

For SSSF process, $\frac{dS_{c.v}}{dt} = 0$ the second law becomes:

$$\sum m^{\cdot}{}_{e}\,s_e - \sum m^{\cdot}{}_{i}\,s_i \geq \iint_{c.s.} \frac{q^{\cdot''}}{T}\,dA$$

For single inlet and single exit, we have

$$m^{\cdot}\,(s_e - s_i) \geq \iint_{c.s.} \frac{q^{\cdot''}}{T}\,dA$$

In addition if the process is adiabatic, it yields:

$$s_e \geq s_i \qquad \text{or} \qquad s_e - s_i \geq 0$$

where equality holds for reversible process

<u>Note</u>: for irreversible adiabatic process, i.e. isentropic process, Se = Si. This is the same as in the case of closed system where we have (equa)

7.12: Reversible SSSF Process

Consider the case of single inlet and single outlet:

1st law: $$q - w = (h_e - h_i) + \frac{V_e^2 - V_i^2}{2} + g(Z_i - Z_e) \qquad (1)$$

2nd law: $$m^{\cdot}(s_e - s_i) \geq \iint\limits_{c.s.} \frac{q^{\cdot''}}{T} dA \qquad (2)$$

We'll consider two situations: adiabatic and isothermal

(i) Adiabatic

reversible and adiabatic ➔ isentropic ➔ Se = Si

Recall: Tds = dh - vdP

For ds = 0, we have:

dh = vdP

or $$\boxed{h_e - h_i = \int_{P_i}^{p_e} vdp} \qquad (3)$$

Substituting (3) into (1) gives:

$$\boxed{w = -\int_{p_i}^{p_e} vdp + \frac{V_e^2 - V_i^2}{2} + g(Z_e - Z_i)} \qquad (4)$$

(ii) Isothermal

Equ. (2) becomes: $$\boxed{m^{\cdot}(s_e - s_i) \geq \frac{1}{T}\iint\limits_{c.s.} \frac{q^{\cdot''}}{T} dA = \frac{Q^{\cdot}_{c.v}}{T}} \qquad (5)$$

or $$s_e - s_i = \frac{q_{c.v}}{T}$$

(*note:* q = heat transfer per unit mass flow)

Recall: Tds = dh – vdP

➔ $$\boxed{T(s_e - s_i) = (h_e - h_i) - \int_{p_i}^{p_e} vdp} \qquad (6)$$

sub. (5) and (6) into (1) gives:

$$T(s_e - s_i) - w = T(s_e - s_i) + \int_{p_i}^{p_e} vdp + \frac{V_e^2 - V_i^2}{2} + g(Z_e - Z_i)$$

➔ $$\boxed{w = -\int_{p_i}^{p_e} vdp + \frac{V_e^2 - V_i^2}{2} + g(Z_i - Z_e)} \qquad (7)$$

Note that Equ. (4) is identical to Equ (7) even though one is for reversible adiabatic process while the other is for reversible isothermal process. Since any reversible process can be constructed, in the limit, from a revies os alternated reversible adiabatic and reversible isothermal process, we may conclude that Equ. (4) is valid for any reversible SSSF process.

Special cases:

(i) w = 0, and v= constant ➔ no work done and incompressible

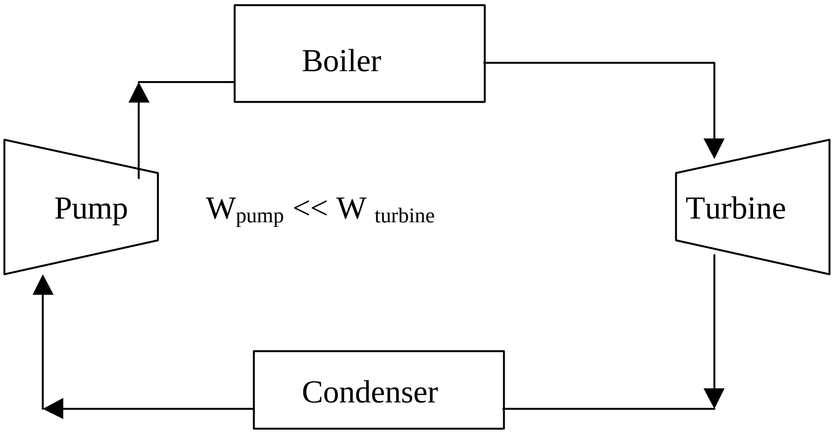
Equ (4) become $\boxed{\frac{P_e}{d} + \frac{V_e^2}{d} + gZ_e = \frac{P_i}{d} + \frac{V_i^2}{2} + gZ_i}$

This is the well know Bernoulli's Equ.

(2) ΔKE, ΔPE are negligible

Equ (4) becomes: $\boxed{w = -\int_{p_i}^{p_e} v\,dp}$

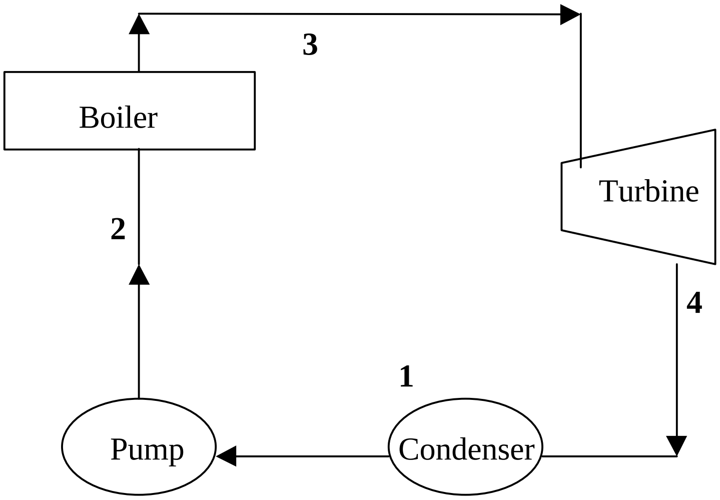
Therefore, for the same $P_e + P_i$, w is much larger for vapor than for liquid since $v_g >> v_l$. Recall for a steam power plant discuss previously, we have

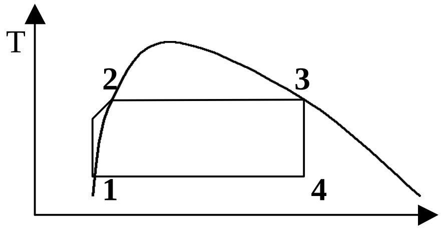


∴ The power input to the pump is much less than the power output of the turbine

7.13: Worked Example # 1

Consider the following Rankine Cycle:





1➔2: isentropic pumping process
2➔3: heat transfer at constant pressure
3➔4: isentropic expansion in the turbine
4➔1: heat transfer at constant pressure

Given: $P_1 = P_4 = 1$ psia
$P_2 = P_3 = 300$ psia
$X_3 = 1$, $X_1 = 0$

Find the efficiency of this Rankine Cycle.

1➔2:

C.∀: Pump

1st law: $q_p - w_p = h_2 - h_1$ where $q_p = 0$

2nd law:
$$s_2 = s_1 \Rightarrow h_2 - h_1 = \int_{p_1}^{p_2} v\,dp = v(p_2 - p_1)$$

Combining 1st and 2nd laws gives: $w_2 = v\,(p_2 - p_1)$
Between 1 to 300 psia $v \approx 0.01614$ ft3 / lbm
Therefore,

$$w_p = 0.01614 \frac{ft^3}{lbm}(1-300)\frac{ft}{in^2}\frac{144 in^2}{ft^2}\frac{1 Btu}{778\, ft}$$

$= \boxed{- 0.9 \text{ Btu / lbm}}$

2➔3:

C.∀: Boiler

1st law: $q_b - w_b = h_3 - h_2$ where $w_b = 0$

From last result, $h_2 = h_1 - w_p$

Now $h_1 = h_f$ (1 psia) = 69.7 Btu / lbm

$h_2 = 69.7 - (-0.9) = 70.6$ Btu / lbm

also $h_3 = h_g$ (300 psia) = 1203.9 Btu / lbm

$q_b = h_3 - h_2 = 1203.9$ Btu /lbm – 70.59 Btu / lbm

$= \boxed{1133.3 \text{ Btu / lbm}}$

3➔4:

C.∀: Turbine

1st law: $q_t - w_t = h_4 - h_3$ where $q_t = 0$

2nd law: $S_3 = S_4$

But $S_3 = S_g$ (300 psia) = 1.5115 Btu / lbm-(R

$S_4 = 1.5115$ Btu / lbm-(R

At $P_4 = 1$ psia $s_f = 0.13266$; $h_f = 69.74$

$s_g = 1.9779$; $h_g = 1105.8$

➔ $$x_4 = \frac{s_4 - s_f}{s_g - s_f} = \frac{1.5115-0.13266}{1.9779-0.13266} = 0.747$$

$h_4 = h_f + x_4 (h_g - h_f)$

$= 69.74 + (0.747) (1105 - 69.74)$

$= 843.8$ Btu / lbm

Applying 1st : $w_t = h_3 - h_4 = 1203.9 - 843.8$

$= \boxed{360.1 \text{ Btu / lbm}}$

4➔1:

C.∀: Condenser

1st law: $q_c - w_c = h_1 - h_4$ where $w_c = 0$

$= 69.7 - 843.8$

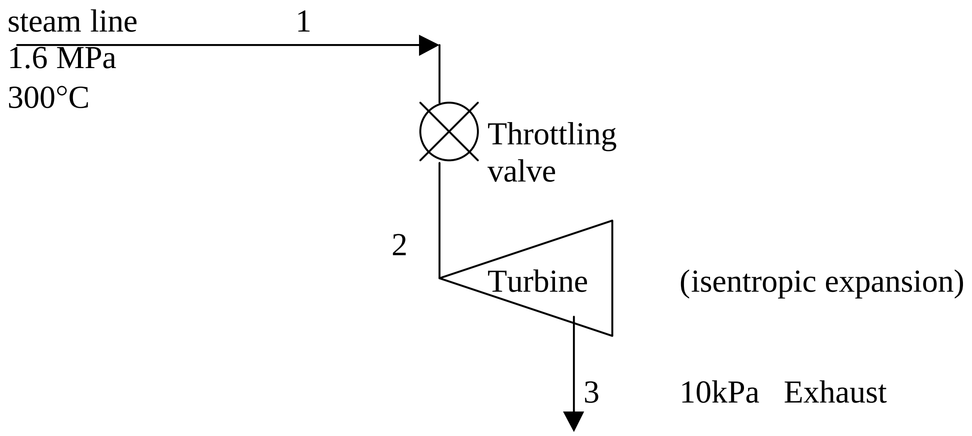
➔ $w_c = -774.1$ Btu / lbm

Finally,

$$h \equiv \frac{|w_{net}|}{|q_H|}\frac{|w_t|-|w_p|}{|q_H|} = \frac{360.1-0.9}{1133.3}$$

$=\boxed{31.6\%}$

7.14 Worked Example # 2 (From Test)



The throttling valve is used to control the work of turbine load output. It is fully open for full load of turbine

(a) Find full load work output of turbine (i.e. valve is fully open)

$S_3 = S_1 = 6.8844$

At P = 10 kPa, $s_f = 0.8493$; $h_f = 191.83$
$s_g = 8.1502$; $h_g = 2584.7$

➔ $$x_3 = \frac{s_3 - s_f}{s_g - s_f} = \frac{6.8844 - 0.6493}{8.1502 - 0.6493} = 0.831$$

➔ $h_3 = h_f + x_3 (h_g - h_f)$
$= 191.83 + 0.831 (2584 – 191.83)$
$= 2180.30$ kJ/ kg

$q_t - w_t = h_3 - h_2$ where $w_t = 0$ and $h_2 = h_1$
$= h_3 - h_1$
$w_t = h_1 - h_3 = 3034.8 - 2180.3$
$= \boxed{854.5 \text{ kJ / kg}}$

(b) Find P_2 which the steam must be throttled to produce 75% of full load output.
$w_{75\%} = (0.75)(854.5) = 640.9$ kJ / kg

➔ $h_3 - h_2 = 640.9$
But $h_2 = h_2 = 3034.8$
➔ $h_3 = h_2 - 640.9$
$= 3034.8 - 640.9$
$= 2393.9$ kJ / kg

Now

$$x_3 = \frac{s_3 - s_f}{s_g - s_f} = \frac{2393.9 - 191.83}{2584.7 - 191.83} = 0.92$$

➔ $$h_3 = h_f + xh_{fg}$$

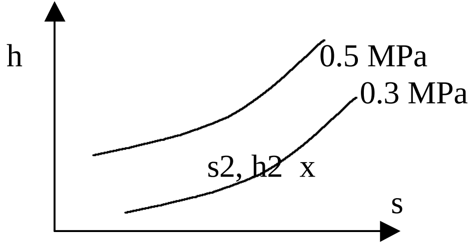
$= 0.6493 + 0.92\ (8.1502 - 0.6493)$
$= 7.552$ kJ / kg-K

Since **2➔3** is isentropic, $s_2 = s_3$

Therefore, we have at **2**

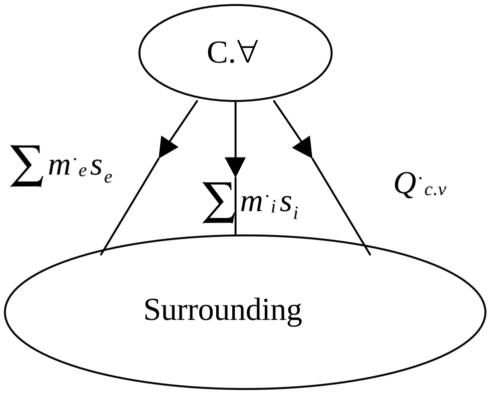
$$\left.\begin{array}{l} s_2 = 7.552 \text{ kJ / kg-K} \\ h_2 = 3034.8 \text{ kJ / kb} \end{array}\right\} \qquad P_2 = 0/368 \text{ Mpa}$$

We have two properties, theoretically we can determine the state, i.e., P_2.



0.3 Mpa < P2 < 0.5 Mpa

7.15 The Principle of Increase of Entropy for a Control Volume



Second law of thermal for C.∀

$$\frac{dS_{c.v}}{dt} + \sum \dot{m}_e s_e - \sum \dot{m}_i s_i \geq \iint_{c.s.} \frac{\dot{q}''}{T} dA \qquad (1)$$

Second law of thermal for the surroundings

$$\frac{dS_{c.v}}{dt} \geq \sum \dot{m}_e s_e - \sum \dot{m}_i s_i - \frac{\dot{Q}_{c.b}}{T_0} \qquad (2)$$

(1) + (2):

$$\frac{dS_{c.v}}{dt} + \frac{dS'_{sum}}{dt} = \iint\limits_{c.s.} \frac{\dot{q}''}{T} dA - \frac{\dot{Q}_{c.v}}{T_0}$$

(i) If heat transferred from the surrounding to the C.∀

$$T_0 > T \Rightarrow \dot{Q}_{c.v} > 0$$

$$\therefore \qquad \iint\limits_{c.s.} \frac{\dot{q}''}{T} dA - \frac{\dot{Q}_{c.v}}{T_0} > 0$$

(ii) If heat loss from the C.∀ to the surrounding

$$T_0 > T \Rightarrow \dot{Q}_{c.v} < 0$$

$$\therefore \qquad \underbrace{\iint\limits_{c.s.} \frac{\dot{q}''}{T} dA}_{\text{Negative quantity}} - \underbrace{\frac{\dot{Q}_{c.v}}{T_0}}_{\text{larger negative quantity}} > 0$$

(because $Q_{c.v} < 0$)

$$\therefore \qquad \boxed{\frac{dS_{c.v}}{dt} + \frac{dS'_{sum}}{dt} \geq 0}$$

7.16 Isentropic Efficiency Device:

The efficiency of a device that involves a process, rather than a cycle, is a composition of the actual performance to that which would be achieved in a related, but well defined ideal process. Thus,

$$h \equiv \frac{Actual-Performance}{Ideal-Performance}$$

Turbine:

$$h = \frac{w_a}{w_s} = \frac{Actual-Work-Output}{Work-Output-Assuming-Isotropic-Process}$$

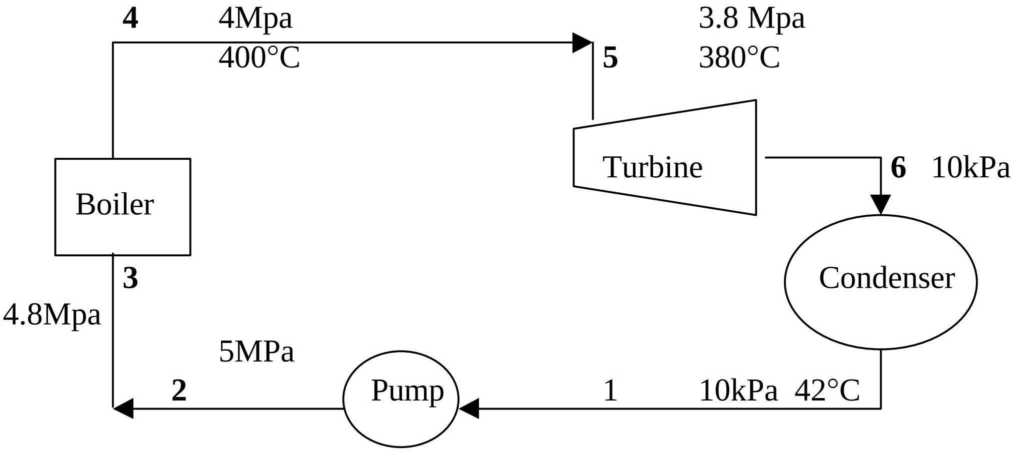
Nozzle:

$$h = \frac{V_a{}^2}{V_s{}^2} = \frac{(Actual-Velocity)^2}{(Isentropic-Velocity)^2}$$

Compressor:

$$h = \frac{w_s}{w_a} = \frac{Work-Input-Assu\min g-Isentropic}{Actual-work-Input-\mathrm{Re}\,ferred}$$

7.17 Worked Example



A steam power plant operates on a cycle with pressures and temperatures as shown above. The efficiency of the turbine is 86% and the efficiency of the pump is 80%. Determine the thermal efficiency of this cycle.

Solution:

C. " : *Turbine*

Inlet state: P_5, T_5 given, i.e state known

Exit state: P_6 known

1st law: $wt = h_5 - h_6$

2nd law: $S_{6s} = S_5$ s: isentropic

$$h = \frac{w_t}{h_5 - h_{6s}} = \frac{h_5 - h_6}{h_5 - h_{6s}}$$

From the steam tables

$h_5 = 3169.1$; $S_5 = 6.7235$

$S_{6s} = S_5 = 6.7235 = s_f + x_{6s}(s_g - s_f)$

$= 0.6493 + x_{6s}\, 7.5009$

➔ $x_{6s} = 0.8098$

$h_{6s} = h_f + x_{6s}(h_g - h_f)$

$= 191.8 + 0.898\,(2392.8)$

$= 2129.5$

w_t = ((h5 - h6s) = 0.86 (3169.1 – 2129.5)
= 894.10 kJ / kg

C. " :   Pump

Inlet state: P_1, T_1 given, state fixed
Exit state: P_2 known
1st law: $w_p = h_2 - h_1$
2nd law: $S_{2s} = S_1$

$$h = \frac{h_{2s}-h_1}{|w_p|} = \frac{h_{2s}-h_1}{h_2-h_1}$$

Since: $S_{2s} = S_1$, $h_{2s} - h_1 = v (P_2 - P_1)$

$$| w_p | = \frac{h_{2s}-h_1}{h} = \frac{v(P_2-P_1)}{h}$$

with v ≈ 0.001009 m3 / kg

$$| w_p | = \frac{(0.001009)(5000-10)}{0.80} = 6.3 kJ / kg$$

(<u>*Note*</u>: $w_p << w_t$)

Therefor, wnet = wt - |wp| = 984.1 – 6.3
= 887.8 kJ / kg

C. " :   Boiler

Inlet state: P_3, T_3 known; state fixed
Exit state: P_4, T_4 known; state fixed
1st law: $q_b = h_4 - h_3$
= 3213.6 – 171.8
= 3041.8 kJ / kg

Finally,

$$h_{th} = \frac{|w_{net}|}{|q_b|} = \frac{887.8}{3041.8} = 292\%$$